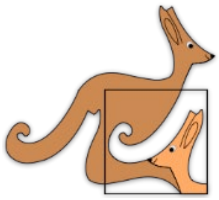


INTERNATIONAL CONTEST-GAME
MATH KANGAROO
CANADA

INSTRUCTIONS
GRADE 11-12

1. You have 75 minutes to solve 30 multiple choice problems. For each problem, decide which answer is correct and fill in (blacken) the oval that has the same letter as the appropriate answer. If you fill in (blacken) more than one oval for a question, your response will be marked as wrong.

1. Record your answers in the response form. Remember that this is the only sheet that is marked, so make sure you have all your answers transferred to that form before giving it back to the contest supervisor.
2. The problems are arranged in three groups. A correct answer of the first 10 problems is worth 3 points. A correct answer of problems 11-20 is worth 4 points. A correct answer of problems 21-30 is worth 5 points. For each incorrect answer, one point is deducted from your score. Each unanswered question is worth 0 points. To avoid negative scores, you start from 30 points. The maximum score possible is 150.
3. The use of external material or aid of any kind is **not permitted**.
4. The figures *are not* drawn to scale. They should be used only for illustration purposes.
5. Remember, you have about 2 to 3 minutes for each problem; hence, if a problem appears to be too difficult, save it for later and move on to another problem.
6. At the end of the allotted time, please **give the response form to the contest supervisor**.
7. Your score and electronic Certificate of Participation will be available in your account after June 1.

Good luck and enjoy!

Canadian Math Kangaroo Contest team

CANADIAN MATH KANGAROO CONTEST PROBLEMS

PART A: EACH CORRECT ANSWER IS WORTH 3 POINTS

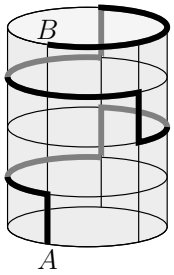
1. What is the value of $\dfrac{7777^2}{5555 \times 2222}$?

(**A**) 1 (**B**) $\frac{7}{10}$ (**C**) $\frac{49}{10}$ (**D**) $\frac{77}{110}$ (**E**) 49

2. Giulia rolls five dice. The sum of the rolled faces is 19.
What is the maximum number of 6 she could have rolled?

(**A**) 0 (**B**) 1 (**C**) 2 (**D**) 3 (**E**) 4

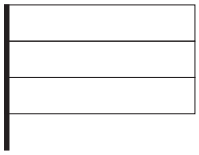
3. A cylindrical can has height 15 cm and the circumference of its base is 30 cm.
An ant walks from point A on the base of the can to point B on the top of the can. Its path is either vertically upwards or horizontally along circular arcs around the can. Its path is shown with a thicker line (black for the path on the front of the can and grey at the back).



What is the length, in cm, of the ant's path?

(**A**) 45 (**B**) 55 (**C**) 60 (**D**) 65 (**E**) 75

4. Emma has four different colour crayons.
She wants to colour the three-striped rectangular flag shown in the figure so that each stripe is a single colour and no two adjacent stripes are the same colour.



In how many ways can she do it?

(**A**) 24 (**B**) 27 (**C**) 32 (**D**) 36 (**E**) 64

5. How many positive integers n have exactly three distinct divisors, if the divisors include 1, 2 and n itself?

(**A**) 0 (**B**) 1 (**C**) 2 (**D**) 3 (**E**) 4

6. How many positive integer roots does the equation $\sqrt{x^2-45}+\sqrt{50-x^2}=x-4$ have?

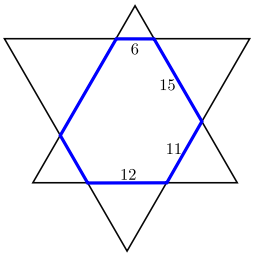
(**A**) 0 (**B**) 1 (**C**) 2 (**D**) 3 (**E**) 4

7. How many pairs of positive integers x and y satisfy the equation $x + 2y = 2^{10}$?

(**A**) $2^9 - 1$ (**B**) 2^9 (**C**) $2^9 + 1$ (**D**) $2^9 + 2$ (**E**) 0

8. Two equilateral triangles are overlapped to form a hexagon with their opposite sides parallel.

We know the length of four sides of this hexagon, as shown in the figure.



What is the perimeter of the hexagon?

(**A**) 64 (**B**) 66 (**C**) 68 (**D**) 70 (**E**) 72

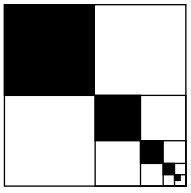
9. Each of the integers from 1 to 9 is to be placed in one of the 9 boxes in the picture so that any three numbers in consecutive boxes add to a multiple of 3.

The numbers 7 and 9 have already been placed.

In how many different ways can the remaining boxes be filled?

(**A**) 9 (**B**) 12 (**C**) 15 (**D**) 18 (**E**) 24

10. A square with area 84 is divided into four squares.
The upper left square is coloured black.
The lower right square is again divided into four squares, and so on.
The process is repeated an infinite number of times.
What is the total area that is coloured black?



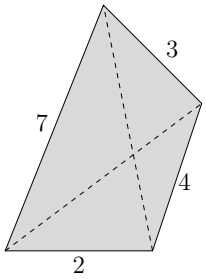
(**A**) 24 (**B**) 28 (**C**) 31 (**D**) 35 (**E**) 42

PART B: EACH CORRECT ANSWER IS WORTH 4 POINTS

11. A triangular pyramid has edges of integer length.

Four of these lengths are as shown in the figure.

What is the sum of the lengths of the other two edges?



(**A**) 9 (**B**) 10 (**C**) 11

(**D**) 12 (**E**) 13

12. For a positive integer n, $n!$ is defined as the product of all integers from 1 to n. For example, $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$. What is the sum of the digits of N if $N! = 6! \cdot 7!$?

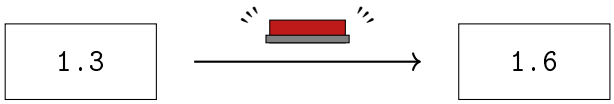
(**A**) 1 (**B**) 2 (**C**) 4 (**D**) 8 (**E**) 9

13. The graphs of the functions $y = x^3 + 3x^2 + ax + 2a + 4$ all pass through the same point in the plane, no matter what value of a is chosen. What is the sum of the coordinates of that point?

(**A**) 2 (**B**) 4 (**C**) 7 (**D**) 8

(**E**) none of the previous

14. Pressing a button increases the number on the screen by the fractional part of that number. For example, if the number on the screen is 1.3, after pressing the button the number on the screen is $1.3 + 0.3 = 1.6$.



How many numbers are there that, if the button is pressed less than four times, would result in the number 10 being displayed?

(**A**) 3 (**B**) 7 (**C**) 8 (**D**) 12 (**E**) 16

15. How many pairs of integers m and n satisfy the inequality $|2m - 2023| + |2n - m| \leq 1$?

(**A**) 0 (**B**) 1 (**C**) 2 (**D**) 3 (**E**) 4

16. There are 23 animals sitting in a row at the cinema. Each animal is either a beaver or a kangaroo. Every animal has at least one neighbour who is a kangaroo.

What is the largest possible number of beavers in the row?

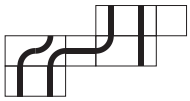
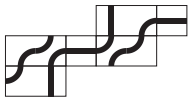
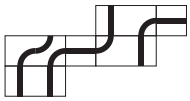
(**A**) 7 (**B**) 8 (**C**) 10 (**D**) 11 (**E**) 12

17. What is the smallest positive integer n such that the interval $\left[\frac{n+8}{2}, \frac{2n+14}{3}\right]$ contains at least four natural numbers?

(**A**) 19 (**B**) 18 (**C**) 17 (**D**) 16

(**E**) none of the previous

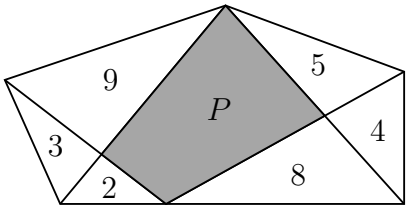
18. Leon has drawn a closed path on a rectangular prism. Which net could show his path?

(**A**)  (**B**)  (**C**)  (**D**)  (**E**) 

19. A pentagon is dissected into parts, as shown.

The numbers inside the triangles indicate their areas.

What is the area P of the shaded quadrilateral?



(**A**) 15 (**B**) $\frac{31}{2}$ (**C**) 16 (**D**) 17 (**E**) 18

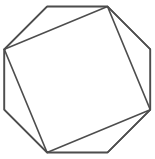
20. How many integers are factors of $2^{20}3^{23}$ but are not factors of $2^{10}3^{20}$?

(**A**) 13 (**B**) 30 (**C**) 273 (**D**) 460

(**E**) none of the previous

PART C: EACH CORRECT ANSWER IS WORTH 5 POINTS

21. From a regular octagon we build a square by joining the vertices as shown.



If the area in cm^2 of the square turns out to be 2 cm^2, what is the area of the octagon in cm^2?

(**A**) $1+\sqrt{2}$ (**B**) $\frac{5}{2}$ (**C**) $2\sqrt{2}$ (**D**) 3 (**E**) $3\sqrt{2}-1$

22. In a bouldering competition, 13 climbers compete in three categories. The score of each competitor is the product of their rankings in the three categories. For example, if one is 4^{th}, 3^{rd} and 6^{th}, their final score is $4 \cdot 3 \cdot 6 = 72$. The higher your score, the lower your overall ranking. Hannah ranks 1^{st} in two of the categories. What is her lowest possible overall ranking?

(**A**) 2^{nd} (**B**) 3^{rd} (**C**) 4^{th} (**D**) 5^{th} (**E**) 6^{th}

23. A spiral of consecutive numbers is created, as shown, starting with 1.

$$\begin{array}{ccccccccc}
\vdots & & 10 & \rightarrow & 11 & \rightarrow & 12 & \rightarrow & 13 \\
\uparrow & & \uparrow & & & & & & \downarrow \\
24 & & 9 & & 2 & \rightarrow & 3 & & 14 \\
\uparrow & & \uparrow & & \uparrow & & \downarrow & & \downarrow \\
23 & & 8 & & 1 & & 4 & & 15 \\
\uparrow & & \uparrow & & & & \downarrow & & \downarrow \\
22 & & 7 & \leftarrow & 6 & \leftarrow & 5 & & 16 \\
\uparrow & & & & & & & & \downarrow \\
21 & \leftarrow & 20 & \leftarrow & 19 & \leftarrow & 18 & \leftarrow & 17
\end{array}$$

The pattern continues indefinitely. In which arrangement will the numbers 625, 626 and 627 appear?

(A)
$$\begin{array}{c} 627 \\ \uparrow \\ 626 \\ \uparrow \\ 625 \end{array}$$

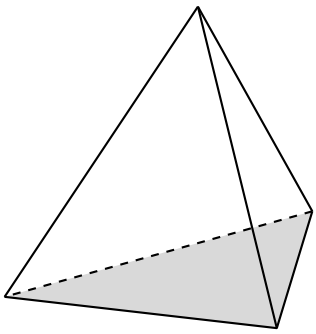
(B)
$$\begin{array}{ccc} 626 & \rightarrow & 627 \\ \uparrow & & \\ 625 & & \end{array}$$

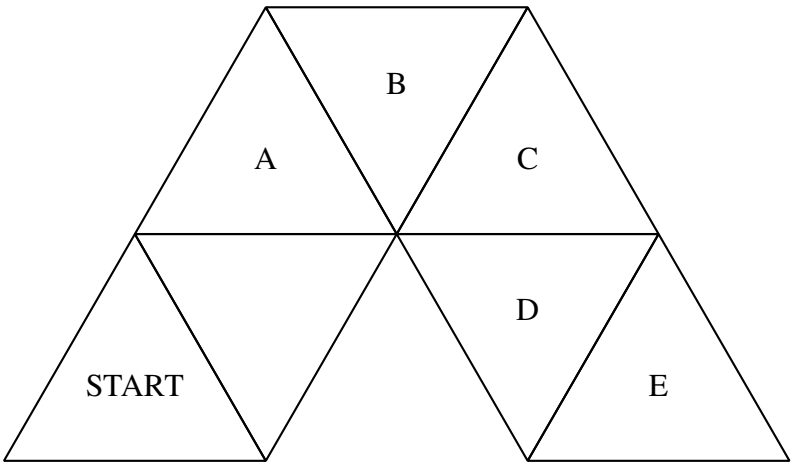
(C) $625 \rightarrow 626 \rightarrow 627$

(D)
$$\begin{array}{ccc} 625 & \rightarrow & 626 \\ & & \downarrow \\ & & 627 \end{array}$$

(E)
$$\begin{array}{c} 625 \\ \downarrow \\ 626 \\ \downarrow \\ 627 \end{array}$$

24. A regular tetrahedron has a shaded face. The shaded face of the tetrahedron is placed on the triangle marked START. The tetrahedron is then rolled from one triangle to another by rotating it around an edge.





On which triangle will the tetrahedron lie next on its shaded face?

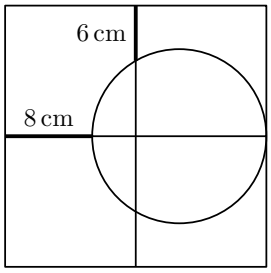
(A) A **(B)** B **(C)** C **(D)** D **(E)** E

25. Part of the fifth degree polynomial shown cannot be seen because of an inkblot. It is known that all five roots of the polynomial are integers.

What is the highest power of $x - 1$ that divides the polynomial?

(A) $(x-1)^1$ **(B)** $(x-1)^2$ **(C)** $(x-1)^3$ **(D)** $(x-1)^4$ **(E)** $(x-1)^5$

26. The large square in the figure is dissected into four equal smaller squares. The circle touches the right hand side of the square at its midpoint.



What is the side-length of the large square?

Note that the figure is not drawn to scale.

(A) 18 cm **(B)** 20 cm **(C)** 24 cm **(D)** 28 cm **(E)** 30 cm

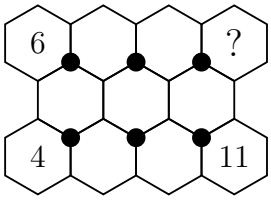
27. What is the greatest common divisor of all numbers of the form

$$n^3(n+1)^3(n+2)^3(n+3)^3(n+4)^3,$$

where n is a non-zero natural number?

(A) $2^9 3^3$ **(B)** $2^3 3^3 5^3$ **(C)** $2^6 3^3 5^3$ **(D)** $2^8 3^2 5^3$ **(E)** $2^9 3^3 5^3$

28. The numbers from 1 to 11 are to be placed in the hexagons so that the sum of the three numbers around each of the six black dots is the same. Three of the numbers have already been placed. What number will be placed in the hexagon with a question mark?



(A) 1 **(B)** 3 **(C)** 5 **(D)** 7 **(E)** 9

29. A teacher wishes to divide a group of ten students into pairs for an activity. However, she doesn't want to put Alice and Bill in the same pair, nor does she want Will and Zoe in the same pair. In how many different ways can she make these pairs?

(A) 300 **(B)** 600 **(C)** 720 **(D)** 750 **(E)** 945

30. The product of six consecutive numbers is a 12-digit number of the form

$$abb\,cdd\,cdd\,abb,$$

where the digits a, b, c and d are themselves four consecutive numbers in some order. What is the value of the digit d?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

CMKC 2023 Grade 11-12 Answers

PART A					
1	A	B	**C**	D	E
2	A	B	**C**	D	E
3	A	B	C	D	**E**
4	A	B	C	**D**	E
5	A	**B**	C	D	E
6	A	**B**	C	D	E
7	**A**	B	C	D	E
8	A	B	C	**D**	E
9	A	B	C	D	**E**
10	A	**B**	C	D	E

PART B					
11	A	B	**C**	D	E
12	**A**	B	C	D	E
13	A	B	C	D	**E**
14	A	B	**C**	D	E
15	A	**B**	C	D	E
16	A	B	C	**D**	E
17	A	B	C	D	**E**
18	A	B	C	**D**	E
19	A	B	**C**	D	E
20	A	B	**C**	D	E

PART C					
21	A	B	**C**	D	E
22	A	**B**	C	D	E
23	A	**B**	C	D	E
24	A	B	C	D	**E**
25	A	B	C	**D**	E
26	**A**	B	C	D	E
27	A	B	C	D	**E**
28	A	B	C	D	**E**
29	A	B	C	**D**	E
30	A	B	**C**	D	E