

**CANADIAN MATH KANGAROO CONTEST PROBLEMS****PART A: EACH CORRECT ANSWER IS WORTH 3 POINTS**

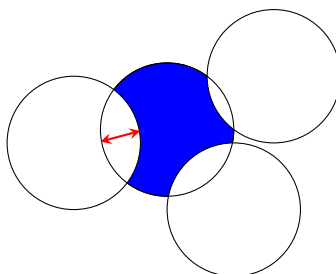
1. How many positive three-digit integers are divisible by 13?

- (A) 68 (B) 69 (C) 70 (D) 76 (E) 77

2. The product of the digits of a 10-digit integer is 15. What is the sum of the digits of this number?

- (A) 8 (B) 12 (C) 15 (D) 16 (E) 20

3. Four circles, each of radius 1, intersect as shown.



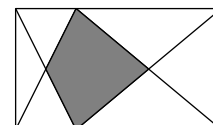
What is the perimeter of the shaded region?

- (A) π (B) π^2 (C) $\frac{3\pi}{2}$ (D) 2π
(E) Some number between $\frac{3\pi}{2}$ and 2π

4. How many real solutions does the equation $(x - 2)^2 + (x + 2)^2 = 0$ have?

- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

5. In the rectangle shown, two points of the two longer sides, which are exactly opposite, are connected with all four vertices.

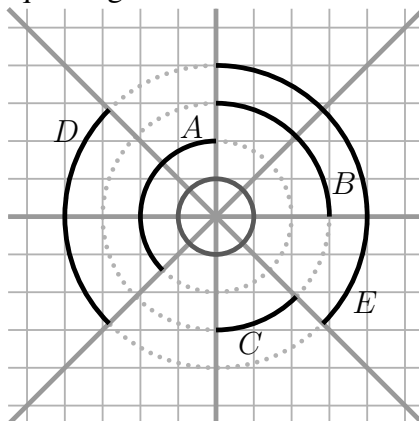


What fraction of the area of the rectangle is shaded?

- (A) $\frac{2}{5}$ (B) $\frac{1}{3}$ (C) $\frac{2}{7}$ (D) $\frac{1}{4}$ (E) $\frac{1}{5}$



6. Four lines intersect forming eight equal angles.



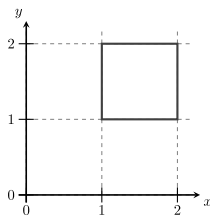
Which black arc has the same length as the smallest circle shown?

- (A) A
- (B) B
- (C) C
- (D) D
- (E) E

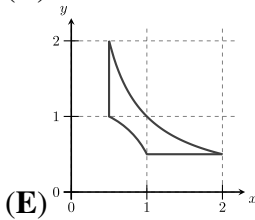
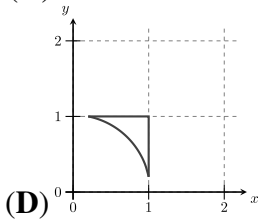
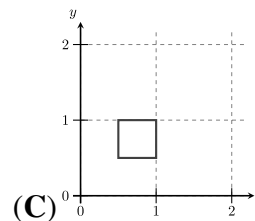
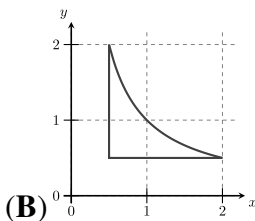
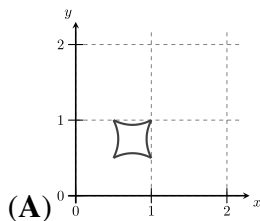
7. Let a , b , and c be non-zero numbers. The numbers $-2a^4b^3c^2$ and $3a^3b^5c^{-4}$ have the same sign. Which of the following is definitely true?

- (A) $ab > 0$
- (B) $b < 0$
- (C) $c > 0$
- (D) $bc > 0$
- (E) $a < 0$

8. A square lies in a coordinate system as shown.



Each point (x, y) on the square is moved to $(\frac{1}{x}, \frac{1}{y})$. What will the resulting figure look like?





9. Mike has marked the points A , B , C and D in this order on a straight line, as shown in the diagram.



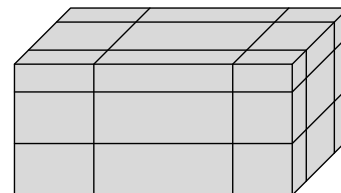
The distance between A and C is 12 cm and between B and D , 18 cm. What is the distance between the midpoint of AB and the midpoint of CD ?

- (A) 15 cm (B) 12 cm (C) 18 cm (D) 6 cm (E) 9 cm
10. Determine the number of different isosceles triangles with integer side lengths, and the longest side (of which there can be more than one) of length 11.

- (A) 5 (B) 6 (C) 16 (D) 17 (E) 18

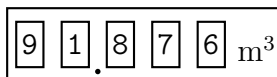
PART B: EACH CORRECT ANSWER IS WORTH 4 POINTS

11. A cuboid of surface area S is cut by six planes as shown. Each plane is parallel to a face, but its distance from the face is random. The cuboid is thus separated into 27 smaller parts.



What, in terms of S , is the total surface area of all 27 smaller parts?

- (A) $2S$ (B) $\frac{5}{2}S$ (C) $3S$ (D) $4S$
- (E) none of the previous
12. When Tony looks at the water meter, he notices that all the digits on the meter are different.

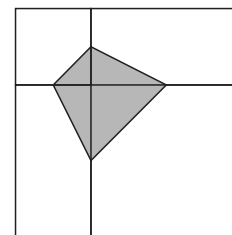


How much water will be used until the next time all the digits on the meter are different?

- (A) 0.006 m^3 (B) 0.034 m^3 (C) 0.086 m^3 (D) 0.137 m^3 (E) 1.048 m^3
13. The largest proper divisor of a number is 2022. (A proper divisor of a number is any divisor of that number other than the number itself.) Which is the second largest divisor of this number?
- (A) 337 (B) 1011 (C) 1348 (D) 2021
- (E) none of the previous



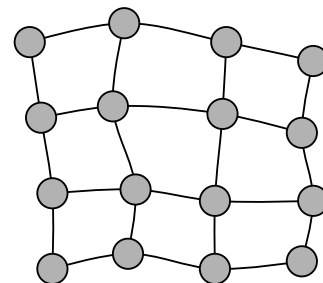
14. A large square is divided into two unequal squares and two congruent rectangles, as shown. The vertices of the shaded quadrilateral are the midpoints of the sides of the two squares.



The area of the shaded quadrilateral is 3.

What is the area of the unshaded part of the large square?

- (A) 12 (B) 15 (C) 18 (D) 21 (E) 24
15. The map shows a region with 16 cities connected by roads. The Government wants to build electricity power plants in some of the cities. Each power plant can provide enough electricity for the city where it is located and any cities connected to that city by a single road.

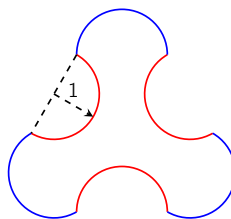


What is the smallest number of power plants that need to be built?

- (A) 3 (B) 4 (C) 5 (D) 6 (E) 7
16. Five numbers have a mean of 24. The mean of the three smallest numbers is 19 and the mean of the three largest numbers is 28. What is the median of the five numbers?

(A) 20 (B) 21 (C) 22 (D) 23 (E) 24

17. Six semicircles, each with radius 1, form a figure similar to a cloverleaf.

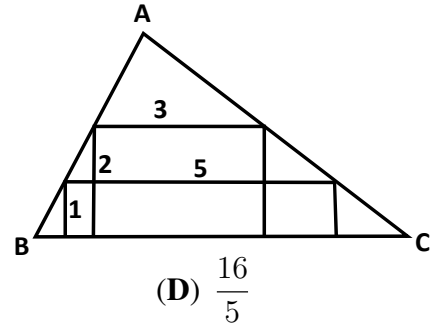


What is its area?

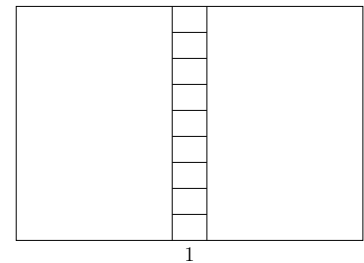
- (A) $2\sqrt{3} + 3\pi$ (B) 3π (C) $\frac{9}{4}\sqrt{3} + \frac{3}{2}\pi$ (D) 9 (E) $6\sqrt{3}$
18. Martina is playing in an 8 player tournament. She knows she will beat everyone, except Ashley who will beat everybody. In the first round, players are organized randomly into four pairs, and the winner of each match proceeds to the second round. In the second round, there are two matches where the opponents are also drawn randomly, and the winners proceed to the final. What is the probability that Martina does not get to the final?

(A) 1 (B) $1/2$ (C) $2/7$ (D) $3/7$ (E) $4/7$

19. Two rectangles are inscribed inside a triangle ABC . The dimensions of the rectangles are 1×5 and 2×3 , respectively, as shown. What is the height of the triangle with base BC ?



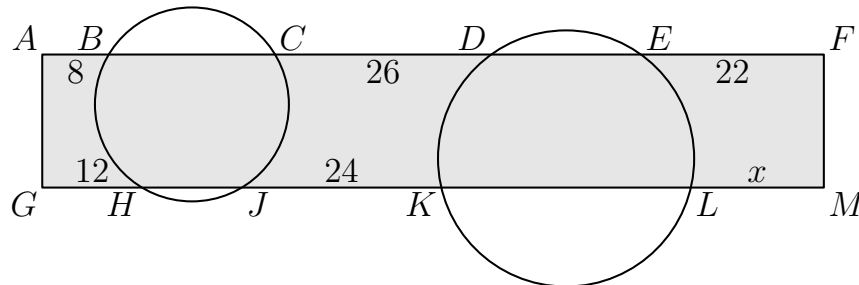
- (A) 3 (B) $\frac{7}{2}$ (C) $\frac{8}{3}$
 (D) $\frac{16}{5}$
 (E) none of the previous
20. A rectangle is divided into 11 smaller rectangles, as shown. All 11 rectangles are similar to the original large rectangle. The orientation of the smallest rectangles is the same as the largest. The length of the base of the smallest rectangles is 1, as shown. What is the perimeter of the large rectangle?



- (A) 20 (B) 24 (C) 27 (D) 30 (E) 36

PART C: EACH CORRECT ANSWER IS WORTH 5 POINTS

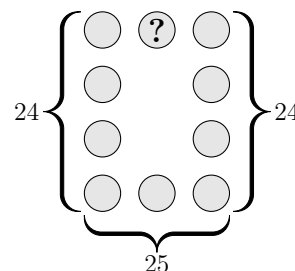
21. How many positive three-digit integers are there that are equal to five times the product of their digits?
- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5
22. Two circles cut a rectangle $AFMG$, as shown.



- The line segments outside the circles have length $AB = 8$, $CD = 26$, $EF = 22$, $GH = 12$, and $JK = 24$. What is the length of LM ?
- (A) 14 (B) 15 (C) 16 (D) 17 (E) 18



23. The numbers 1 to 10 are placed, once each, in the circles of the figure. The sum of the numbers in the left column is 24; the sum of the numbers in the right column is also 24 and the sum of the numbers in the bottom row is 25.



What number is in the circle containing the question mark?

- (A) 2 (B) 4 (C) 5 (D) 6
(E) none of the previous
24. The coefficients of a polynomial P are single digit natural numbers (that is, any of 0, 1, 2, ..., 9). If $P(\sqrt{10}) = 12 + 34\sqrt{10}$, what is the value of $P(10)$?
- (A) 46 (B) 352 (C) 2022 (D) 3142
(E) none of the previous
25. The vertices of a 20-gon are numbered from 1 to 20 in such a way that the numbers of adjacent vertices differ by either 1 or 2. The sides of the 20-gon whose ends differ by only 1 are colored red. How many red sides are there?

- (A) 1 (B) 2 (C) 5 (D) 10
(E) none of the previous

26. Let N be a positive integer.

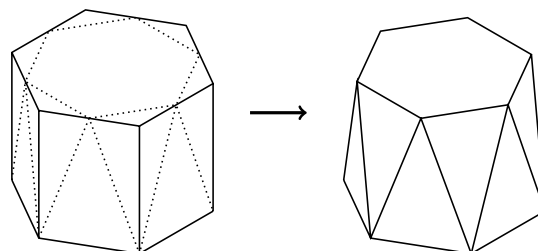
How many integers are there between $\sqrt{N^2 + N + 1}$ and $\sqrt{9N^2 + N + 1}$?

- (A) $N + 1$ (B) $2N - 1$ (C) $2N$ (D) $2N + 1$ (E) $3N$
27. In a sequence, a_n , the first term, a_1 , is between 0 and 1.
For all $n \geq 1$,
 $a_{2n} = a_2 \cdot a_n + 1$, and
 $a_{2n+1} = a_2 \cdot a_n - 2$.
- Given that $a_7 = 2$, what is the value of a_2 ?

- (A) Equal to a_1 (B) 2 (C) 3 (D) 4 (E) 5



28. A regular hexagonal prism has its top corners shaved off, as shown. The top face becomes a smaller regular hexagon and the 6 rectangular faces around the middle become 12 isosceles triangles of two different sizes.



What fraction of the volume of the original prism has been removed?

- (A) $\frac{1}{12}$ (B) $\frac{1}{6}$ (C) $\frac{1}{4\sqrt{3}}$ (D) $\frac{1}{6\sqrt{2}}$ (E) $\frac{1}{6\sqrt{3}}$
29. A football match between teams from North Berracan and South Berracan is played in a stadium that has a rectangular array of seats for the spectators. There are 11 North Berracan supporters in each row, and 14 South Berracan supporters in each column. This leaves 17 empty seats. What is the smallest possible number of seats in the stadium?
- (A) 500 (B) 660 (C) 690 (D) 840 (E) 994
30. Elisapie was using an app to track her progress while running in a 5 km race. As shown in the diagram, Elisapie ran the first kilometer in 10 minutes and 37 seconds, the second kilometer in 10 minutes and 49 seconds, etc. After finishing the race, Elisapie realised that the app did not register the time for the fourth kilometer. (Note that the diagram is not drawn to scale.)

Time per Kilometer

KM	TIME	
1	10:37	
2	10:49	
3	10:41	
4	?	
5	10:25	

Given that exactly three of the five registered times were below the average time per kilometer, which of the following could be Elisapie's time for the fourth kilometer of the race?

- (A) 10:50 (B) 11:25 (C) 11:35 (D) 11:45 (E) 11:55

**CMKC 2022 Grade 11-12 Answers**

PART A						PART B						PART C					
1	A	<u>B</u>	C	D	E	11	A	B	<u>C</u>	D	E	21	<u>A</u>	B	C	D	E
2	A	B	C	<u>D</u>	E	12	A	B	C	<u>D</u>	E	22	A	B	<u>C</u>	D	E
3	A	B	C	<u>D</u>	E	13	A	B	<u>C</u>	D	E	23	A	B	C	D	<u>E</u>
4	<u>A</u>	B	C	D	E	14	A	B	C	<u>D</u>	E	24	A	B	C	<u>D</u>	E
5	A	B	C	<u>D</u>	E	15	A	<u>B</u>	C	D	E	25	A	<u>B</u>	C	D	E
6	A	B	C	<u>D</u>	E	16	A	<u>B</u>	C	D	E	26	A	B	<u>C</u>	D	E
7	A	B	C	D	<u>E</u>	17	A	B	C	D	<u>E</u>	27	A	B	C	<u>D</u>	E
8	A	B	<u>C</u>	D	E	18	A	B	C	<u>D</u>	E	28	<u>A</u>	B	C	D	E
9	<u>A</u>	B	C	D	E	19	A	<u>B</u>	C	D	E	29	A	<u>B</u>	C	D	E
10	A	B	<u>C</u>	D	E	20	A	B	C	<u>D</u>	E	30	A	<u>B</u>	C	D	E