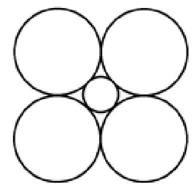
International Contest-Game
MATH KANGAROO

Part A: Each correct answer is worth 3 points.

1. Which of the following numbers is the largest?
(A) 2013 (B) 2^{0+13} (C) 20^{13} (D) 201^3 (E) 20×13

2. Four circles of radius 1 are touching each other and a smaller circle as seen in the picture. What is the radius of the smaller circle?



(A) $\sqrt{2}-1$ (B) $\frac{1}{2}$ (C) $\frac{\sqrt{3}}{4}$ (D) $\frac{3}{4}$ (E) $\frac{7}{16}$

3. A three-dimensional object bounded only by polygons is called a polyhedron. What is the smallest number of polygons that can bind a polyhedron, if we know that one of the polygons has 12 sides?
(A) 12 (B) 13 (C) 14 (D) 16 (E) 24

4. The cube root of 3^{3^3} is equal to
(A) 3^3 (B) $3^{3^3}-1$ (C) 3^{2^3} (D) 3^{3^2} (E) $(\sqrt{3})^3$

5. The year 2013 has the property that its number is made up of the consecutive digits 0, 1, 2 and 3. How many years have passed since the last time a year was made up of four consecutive digits?
(A) 467 (B) 527 (C) 581 (D) 693 (E) 990

6. Let f be a linear function for which $f(2013) - f(2001) = 100$. What is $f(2031) - f(2013)$?
(A) 75 (B) 100 (C) 120 (D) 150 (E) 180

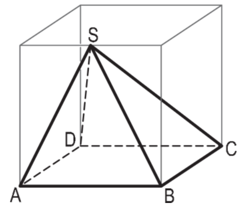
7. Given that $2 < x < 3$, how many of the following statements are true?

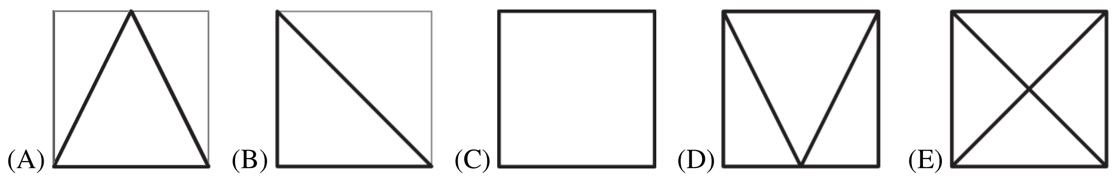
$4 < x^2 < 9$ $\quad 4 < 2x < 9$ $\quad 6 < 3x < 9$ $\quad 0 < x^2 - 2x < 3$

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

8. Six superheroes capture 20 villains. The first superhero captures one villain, the second captures two villains and the third captures three villains. The fourth superhero captures more villains than any of the other five. What is the smallest number of villains the fourth superhero must have captured?
(A) 7 (B) 6 (C) 5 (D) 4 (E) 3

9. In the cube to the right you see a solid, non-transparent pyramid $ABCDS$ with base $ABCD$, whose vertex S lies exactly in the middle of an edge of the cube. You look at this pyramid from above, from below, from behind, from ahead, from the right and from the left. Which view does not arise?



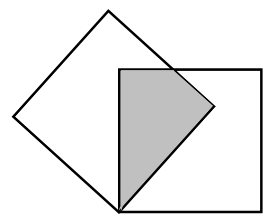


10. When a certain solid substance melts, its volume increases by $\frac{1}{12}$. By how much does its volume decrease when it solidifies again?

(A) $\frac{1}{10}$ (B) $\frac{1}{11}$ (C) $\frac{1}{12}$ (D) $\frac{1}{13}$ (E) $\frac{1}{14}$

<u>Part B: Each correct answer is worth 4 points.</u>

11. The diagram shows two squares of equal side length placed so that they overlap. The squares have a common vertex and the sides make an angle of 45 degrees with each other, as shown. What is the area of the overlap as a fraction of the area of one square?



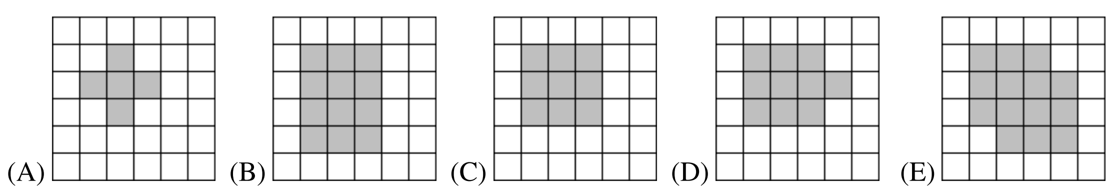
(A) $\frac{1}{2}$ (B) $\frac{1}{\sqrt{2}}$ (C) $1-\frac{1}{\sqrt{2}}$

(D) $\sqrt{2}-1$ (E) $\frac{\sqrt{2}-1}{2}$

12. How many positive integers n exist such that both $\frac{n}{3}$ and $3n$ are three-digit integers?

(A) 12 (B) 33 (C) 34 (D) 100 (E) 300

13. A circular carpet is placed on a floor of square tiles. All the tiles which have more than one point in common with the carpet are marked grey. Which of the following is an impossible outcome?



14. Consider the following statement about a function f on the set of integers: "For any even x, $f(x)$ is even." What would be the negation of this proposition?

(A) For any even x, $f(x)$ is odd (B) For any odd x, $f(x)$ is even

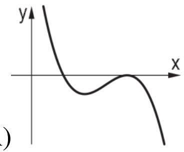
(C) For any odd x, $f(x)$ is odd (D) There exists an even number x such that $f(x)$ is odd

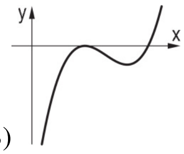
(E) There exists an odd number x such that $f(x)$ is odd

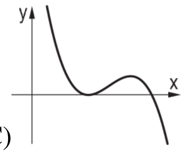
15. How many pairs (x,y) of positive integers satisfy the equation $x^2y^3 = 6^{12}$?

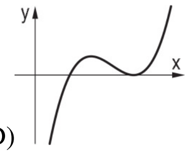
(A) 6 (B) 8 (C) 10 (D) 12 (E) Another number.

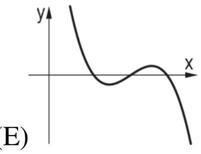
16. Given a function $W(x) = (a-x)(b-x)^2$, where $a < b$. Its graph is in one of the following figures. In which one?

(A) 

(B) 

(C) 

(D) 

(E) 

17. Consider a rectangle, one of whose sides has a length of 5. The rectangle can be cut into a square and a rectangle, one of which has the area 4. How many such rectangles exist?

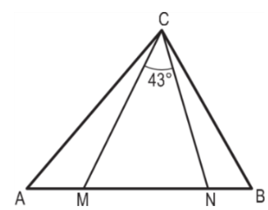
(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

18. Assume that $x^2 - y^2 = 84$, where x and y are positive integers. How many values may the expression $x^2 + y^2$ have?

(A) 1 (B) 2 (C) 3 (D) 5 (E) 6

19. In the triangle ABC the points M and N on the side AB are such that $AN = AC$ and $BM = BC$. Find $\angle ACB$ if $\angle MCN = 43°$.

(A) 86° (B) 89° (C) 90°
(D) 92° (E) 94°



20. A box contains 900 cards numbered from 100 to 999. Any two cards have different numbers. François picks some cards and determines the sum of the digits on each of them. At least how many cards must he pick in order to be certain to have three cards with the same sum?

(A) 51 (B) 52 (C) 53 (D) 54 (E) 55

<u>Part C: Each correct answer is worth 5 points.</u>

21. How many pairs (x,y) of integers with $x \le y$ exist such that their product equals 5 times their sum?

(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

22. Let $f(x), x \in R$ be the function defined by the following properties: f is periodic with period 5 and $f(x) = x^2$ when $x \in [-2,3)$. What is $f(2013)$?

(A) 0 (B) 1 (C) 2 (D) 4 (E) 9

23. We have many white cubes and many black cubes, all of the same size. We want to build a rectangular prism composed by exactly 2013 of these cubes so that they are placed alternating a white cube and a black cube in all directions. If we start putting a black cube in one of the eight corners of the prism, how many black squares will we see on the exterior surface of the solid?

(A) 887 (B) 888 (C) 890 (D) 892
(E) It depends on the dimensions of the prism

24. How many solutions (x,y), where x and y are real numbers, does the equation $x^2 + y^2 = |x| + |y|$ have?

(A) 1 (B) 5 (C) 8 (D) 9 (E) Infinitely many.

25. There are 2013 points marked inside a square. Some of them are connected to the vertices of the square and with each other so that the square is divided into non-overlapping triangles. All marked points are vertices of these triangles. How many triangles are formed this way?

(A) 2013 (B) 2015 (C) 4026 (D) 4028 (E) impossible to determine

26. There are some straight lines drawn on the plane. Line a intersects exactly three other lines and line b intersects exactly four other lines. Line c intersects exactly n other lines, with $n \neq 3, 4$.

Determine the number of lines drawn on the plane.

(A) 4 (B) 5 (C) 6 (D) 7 (E) Another number.

27. The sum of the first n positive integers is a three-digit number in which all of the digits are the same. What is the sum of the digits of n?

(A) 6 (B) 9 (C) 12 (D) 15 (E) 18

28. On the island of Knights and Knaves there live only two types of people: Knights (who always speak the truth) and Knaves (who always lie). I met two men who lived there and asked the taller man if they were both Knights. He replied, but I could not figure out what they were, so I asked the shorter man if the taller was a Knight. He replied, and after that I knew which type they were. Were the men Knights or Knaves?

(A) They were both Knights.

(B) They were both Knaves.

(C) The taller was a Knight and the shorter was a Knave.

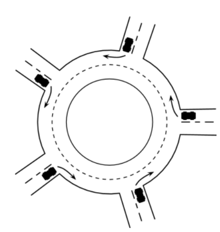
(D) The taller was a Knave and the shorter was a Knight.

(E) Not enough information is given.

29. Julian has written an algorithm in order to create a sequence of numbers as $a_1 = 1$, $a_{m+n} = a_m + a_n + mn$, where m and n are natural numbers. Find the value of a_{100}.

(A) 100 (B) 1000 (C) 2012 (D) 4950 (E) 5050

30. The roundabout shown in the picture is entered by 5 cars at the same time, each one from a different direction. Each of the cars drives less than one round and no two cars leave the roundabout in the same direction. How many different combinations are there for the cars leaving the roundabout?



(A) 24 (B) 44 (C) 60 (D) 81 (E) 120