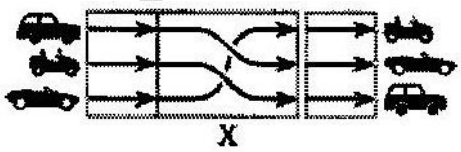
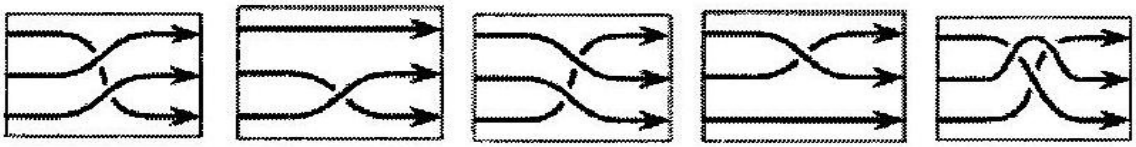
International Contest-Game
MATH KANGAROO
Canada, 2007

Grade 11 and 12

Part A: Each correct answer is worth 3 points.

1. Mike is building a race track. He wants the cars to start the race in the order presented on the left, and to end the race in the order presented to the right. Which of the elements given below should Mike use to replace the element X ?



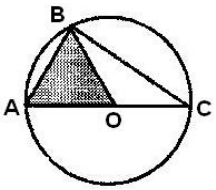


A) B) C) D) E)

2. What is the value of $\dfrac{\sin 1°}{\cos 89°}$?

A) 0 B) tan 1° C) 1/ tan1° D) 1/89 E) 1

3. On the figure, the triangle ABC is inscribed in a circle with a centre O. The shaded area is equal to $\sqrt{3}$. What is the area of the triangle ABC?

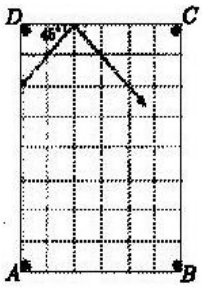


A) $2\sqrt{3}$ B) 2 C) 5 D) 4 E) $4\sqrt{3}$

4. At the entrance examination to a university, a student must answer at least 80% of the questions correctly. So far, Peter has worked on 15 questions. He did not know the answer to 5 of them, but he is sure that he has answered the other 10 questions correctly. If he answers all the remaining questions in test correctly, he will pass the test at exactly 80%. How many questions are there in the test?

A) 20 B) 25 C) 30 D) 35 E) 40

5. A billiard ball is hit from a point on the vertical board of the table (close to the pocket D), as shown in the diagram. It meets the horizontal board at an angle of 45°, and continues following the direction of the arrow. Into which pocket will the ball fall?



A) A B) B C) C D) D E) neither of the pockets

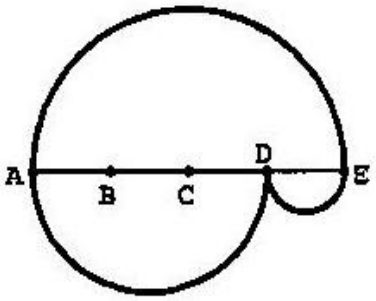
6. Some historians claim that the ancient Egyptians used a string with 2 knots to construct a right angle. If the length of the string is 12 m and one of the knots is at the point X 3 m from one of the ends, at what distance from the other end of the string should the second knot be put in order to obtain a right angle at X?

A) 3 B) 4 C) 5 D) 6 E) None of these

7. An island is inhabited by knights and liars. Each knight always tells the truth and each liar always lies. Once an islander A, when asked about himself and another islander B, claimed that at least one of A and B is a liar. Which of the following sentences is true?

A) A is not able to make the above statement.
B) Both are liars.
C) Both are knights.
D) A is a liar while B is a knight.
E) B is a liar while A is a knight.

8. In the figure, AE is divided into four equal parts. Three semicircles are constructed taking AE, AD, and DE as diameters, and creating two paths from A to E, as shown. What is the ratio of the length of the upper path to the length of the lower path?



A) 1:2 B) 2:3 C) 2:1 D) 3:2 E) 1:1

Part B: Each correct answer is worth 4 points.

9. Given a square ABCD with side 1, all squares are drawn that share at least two vertices with ABCD. What is the area of the region obtained by stretching a rubber band around the squares?

A) 5 B) 6 C) 7 D) 8 E) 9

10. Angle β is 25 % less than angle χ and 50 % greater than angle α. Which of the following is true about angle χ?

A) It is 25% greater than α B) It is 50% greater than α C) It is 75% greater than α D) It is 100% greater than α E) It is 125% greater than α

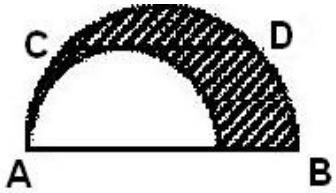
11. Given $2^{x+1} + 2^x = 3^{y+2} - 3^y$, where x and y are integers. What is the value of x?

A) 0 B) 3 C) -1 D) 1 E) 2

12. What is the value of $\cos 1° + \cos 2° + \cos 3° + \ldots + \cos 358° + \cos 359°$?

A) 1 B) π C) 0 D) 10 E) -1

13. Two semicircles are constructed, as shown in the figure. The chord CD is parallel to the diameter AB of the greater semicircle and touches the smaller semicircle. If the length of CD is 4, what is the area of the shaded region?



A) π B) 1.5π C) 2π D) 3π E) more information needed

14. The sum of five consecutive integers is equal to the sum of the next three consecutive integers. What is the greatest of these eight numbers?

A) 4 B) 8 C) 9 D) 11 E) something else

15. Thomas was born on his mother's 20th birthday, and so they share birthdays. How many times will Thomas's age be a divisor of his mother's age if they both live long lives?

A) 4 B) 5 C) 6 D) 7 E) 8

16. Consider a sphere of radius 3 with centre at the origin of a Cartesian co-ordinate system. How many points on the surface of this sphere have integer co-ordinates?

A) 30 B) 24 C) 12 D) 6 E) 3

Part C: Each correct answer is worth 5 points.

17. Which of the following numbers cannot be written as $x + \sqrt{x}$, if x is an integer number?

A) 870 B) 110 C) 90 D) 60 E) 30

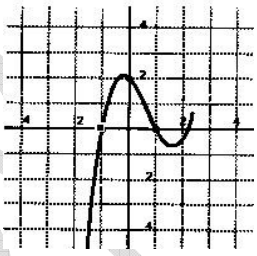
18. If $f(x) = \dfrac{2x}{3x+4}$ and $f(g(x)) = x$, what is the equation of the function $g(x)$?

A) $g(x) = \dfrac{3x+4}{2x}$ B) $g(x) = \dfrac{3x}{2x+4}$ C) $g(x) = \dfrac{2x+4}{4x}$ D) $g(x) = \dfrac{4x}{2-3x}$ E) Other answer

19. What is the measure of the acute angles of a rhombus, if its side is the geometrical mean of the diagonals? *(Note: The number* $C = \sqrt{A \times B}$ *is called Geometrical mean of the numbers* A and B).

A) 15° B) 30° C) 45° D) 60° E) 75°

20. The graphic on the right is a piece of the graph of the function $f(x) = ax^3 + bx^2 + cx + d$. It passes through the points (-1, 0), (0, 2) and (1, 0). What is the value of b?



A) -4 B) -2 C) 0 D) 2 E) 4

21. For how many real numbers a does the quadratic equation $x^2 + ax + 2007 = 0$ have two integer roots?

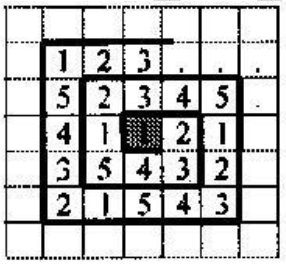
A) 3 B) 4 C) 6 D) 8 E) another answer

22. On a party, five friends are going to give each other gifts in such a way that everybody gives one gift and receives one gift (of course, no one should receive their own gift). In how many ways is this possible?

A) 5 B) 10 C) 44 D) 50 E) 120

23. What is the sum $\frac{1}{2\sqrt{1}+1\sqrt{2}} + \frac{1}{3\sqrt{2}+2\sqrt{3}} + \frac{1}{4\sqrt{3}+3\sqrt{4}} + \ldots + \frac{1}{100\sqrt{99}+99\sqrt{100}}$?

A)999/1000 B) 99/100 C) 9/10
D) 9 E)1



24. The digits of the sequence 123451234512345… fill the cells in the table in a spiral-like manner beginning from the marked cell (see the figure). Which digit is written on the cell placed 100 cells above the marked one?

A) 1 B) 2 C) 3 D) 4 E) 5

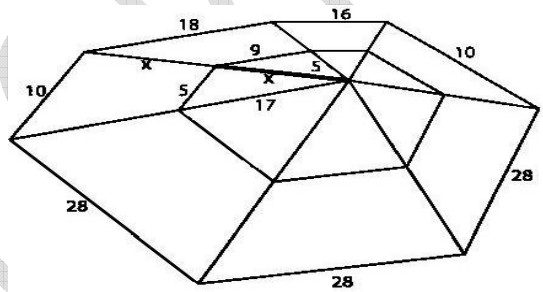
End of Problems

Bonus Problems

Bonus 1: *The increasing sequence 1, 3, 4, 9, 10, 12, 13, … includes all the powers of 3 and all the numbers that can be written as the sum of different powers of 3. What is the 100^{th} element of the sequence?*

A) 150 *B) 981* *C) 1234* *D) 2401* *E) 3^{100}*

Bonus 2: *A mathematically skilled spider spins a web and some of the strings have lengths as shown in the picture. If x is an integer, determine the value of x.*



A) 11 *B)*13 *C)* 15 *D)* 17 *E) 19*

Bonus 3: *Ann, Belinda and Charles are throwing a die. Ann wins if she throws a 1, 2, or 3; Belinda wins if she throws a 4 or 5; Charles wins if he throws a 6. The die rotates from Ann to Belinda to Charles to Ann, etc. , until one player wins. Calculate the probability that Charles wins.*

A) 1/6 *B)*1/8 *C)* 1/11 *D)* 1/13 *E) It is impossible for Charles to win*