International Contest-Game MATH KANGAROO
Canada, 2007

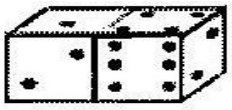
Grade 9 and 10

Part A: Each correct answer is worth 3 points.

1. Anh, Ben and Chen have 30 balls altogether. If Ben gives 5 balls to Chen, Chen gives 4 balls to Anh and Anh gives 2 balls to Ben, then the three boys will have an equal number of balls. How many balls did Anh have at first?

A) 8 B) 9 C) 11 D) 13 E) 15

2. A regular die has a total of 7 points on any two of its opposite faces. On the figure, two regular dice are placed, as shown. What is the sum of the points on all invisible faces of the dice?



A) 15 B) 12 C) 7 D) 27 E) another answer

3. When announcing the results of a raffle, the moderator said: *"The winning tickets are those, which contain a number with at least 5 digits such that at most three of digits are greater than 2."* Subsequently, the speaker drew tickets with numbers 1022, 22222, 102334, 213343, 3042531. How many of them were winning ones?

A) 1 B) 2 C) 3 D) 4 E) 5

4. In the triangle ABC, D is the midpoint of AB, E is the midpoint of DB, F is the midpoint of BC. If the area of triangle ABC is 96, what is the area of triangle AEF?

A) 16 B) 24 C) 32 D) 36 E) 48

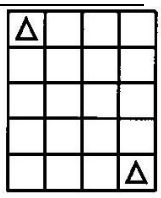
5. Frida has 2007 marbles kept in three bags, A, B, and C. Each bag contains the same number of marbles. If Frida moves 2/3 of the marbles in bag A to bag C, what will be the ratio between the number of marbles in bag A and bag C?

A) 1:2 B) 1:3 C) 2:3 D) 1:5 E) 3:2

6. An international organisation has 32 members. It is predicted that the organisation will increase the number of its members by 50% each year. How many members will the organisation have in three years?

A) 182 B) 128 C) 108 D) 96 E) 80

7. In one move, the King can go to any adjacent square, along a row, column, or diagonal. How many routes with the minimum number of moves are there for the king to travel from the top left square to the bottom right square on the grid?



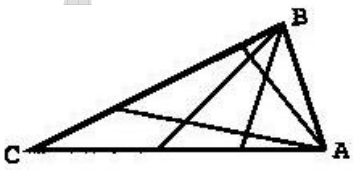
A) 1 B) 4 C) 7 D) 20 E) 35

8. What is the least possible value of the expression 2007 – KAN – GA – ROO, if it is given that each letter represents a digit (different letters represent different digits and the same letters – equal digits)?

A) 100 B) 110 C) 112 D) 119 E) 129

Part B: Each correct answer is worth 4 points.

9. On the diagram, the triangle ABC is divided into nine non-overlapping sections by drawing two lines from each of the vertices A and B. How many non-overlapping sections will there be if four lines are drawn from each of these vertices?

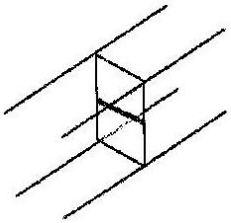


A) 16 B) 25 C) 30 D) 42 E) 49

10. An island is inhabited by liars and nobles (the liars always tell lies and the nobles always tell the truth). One day, 12 islanders, among them both liars and nobles, gathered together and issued a few statements. Two people said: "Exactly two people among us twelve are liars". Another four people said: " Exactly four people among us twelve are liars". The remaining six people said: "Exactly six people among us twelve are liars". How many liars were there?

A) 10 B) 8 C) 6 D) 4 E) 2

11. A hallway is sagged on the right side. As a consequence, the profile is no longer a rectangle but a parallelogram. There is a door halfway through the hallway. The door has two sections, which can be opened separately. Where should they put the hinges?



A) both left B) both right C) above left, below right D) below left, above right E) The door can never open properly

12. A 10 m rope is fastened to one of the corners of a house, which has the form of a rectangle, 6 m long and 4 m wide. A dog is fastened to the rope. What is the perimeter of the region that the dog can access?

A) 20π B) 22π C) 40π D) 88π E) 100π

13. A trapezoid is formed by removing a corner from an equilateral triangle. Two copies of this trapezoid are placed side by side to form a parallelogram. The perimeter of the parallelogram exceeds the perimeter of the original triangle by 10 cm. What is the perimeter of the original equilateral triangle?

A) 10 cm B) 30 cm C) 45 cm D) 60 cm E) more information needed

14. A sequence of letters KANGAROOKANGAROO…KANGAROO contains 20 words KANGAROO. First, all letters in the odd positions of the sequence were erased. Then, in the sequence obtained, once again all the letters in the odd places were erased, and so on, At the very end, only one letter remained. What is this letter?

A) K B) A C) N D) G E) O

15. What percent of the natural numbers from 1 to 10000 are perfect squares?
(*Perfect square is a number that can be presented as a square of a natural number, for instance* $100 = 10^2$).

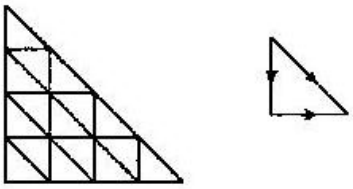
A) 1% B) 1.5% C) 2% D) 2.5% E) 5%

16. The number 8^8 is obtained from the number 4^4 by raising this number by the power of **n.** What is the number **n**?

A) 2 B) 3 C) 4 D) 8 E) 16

Part C: Each correct answer is worth 5 points.

17. A map of a neighbourhood is shown on the figure. The neighbourhood has a form of a right-angled triangle. All streets are shown on the map. They divide the neighbourhood into blocks that also are right-angled triangles. Alex wants to drive from the top endpoint of the neighbourhood to the rightmost endpoint of the neighbourhood. The traffic regulations only allow going down (vertically), right (horizontally), or down by a "hypotenuse" of a block. From how many different routes can Alex choose?

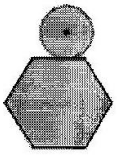


A) 16 B) 27 C) 64 D) 90 E) 111

18. The first digit of a 4 – digit number is equal to the number of zeroes in this number. The second digit of the number is equal to the number of digits 1, the third digit is equal to the number of the digits 2, and the fourth digit represents the number of the digits 3 in this number. How many numbers have this property?

A) 0 B) 2 C) 3 D) 4 E) 5

19. A coin with a diameter 1 cm rolls around the outside of a regular hexagon with a side length of 1 cm, as shown. What is the length (in cm) of the path traced out by the centre of the coin after one complete rotation around the hexagon?



A) $6 + \pi/2$ B) $6+\pi$ C) $12 +\pi$ D) $6+2\pi$ E) $12 +2\pi$

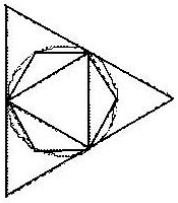
20. Let A be the least natural number with the following property: $10\times A$ is a perfect square (a second exponent of a natural number) and $6\times A$ is a perfect cube (a third exponent of a natural number). How many positive divisors does the number A have?

A) 30 B) 40 C) 54 D) 72 E) 96

21. On a party, five friends are going to give each other gifts in such a way that everybody gives one gift and receives one gift (of course, no one should receive their own gift). In how many ways is this possible?

A) 5 B) 10 C) 44 D) 50 E) 120

22. An equilateral triangle and a regular hexagon are inscribed in a circle, which itself is inscribed in a larger equilateral triangle (see the figure). S1 denotes the area of the greater triangle, S2 is the area of the smaller triangle, and S3 is the area of the hexagon. Which of the following equalities is true?



A) $S3 = \sqrt{S1 \times S2}$

B) $S3 = \dfrac{S1 + S2}{2}$

C) $S1 = S2 + S3$

D) $S3 = \sqrt{S1^2 \times S2^2}$

E) $S1 = S3 + 3 \times S2$

23. It is 21:00 hours (military time). I am driving at 100 km/h. With this speed I have enough gas to pass 80 km. The nearest gas station is 100 km away. The amount of gas my car uses per 1 km is proportional to the velocity of the car. I want to reach the gas station as soon as possible. At what time, at the earliest, can I arrive at the gas station?

A) 22:12 B) 22:15 C) 22:20 D) 22:25 E) other answer

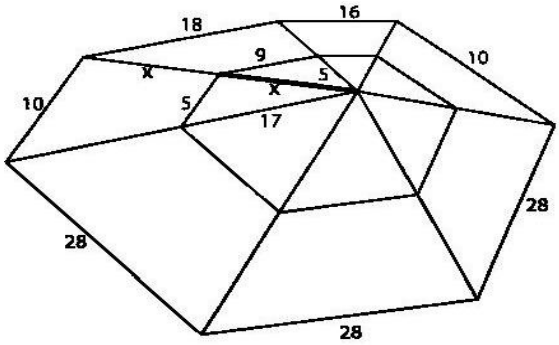
24. There are three green, three red, three blue and three yellow cards in a box. Each of the three cards from each colour is numbered by a number 1, 2, or 3 (e.g. there is one green card with a number 1, one green card with a number 2, one green card with a number 3, etc.). We take randomly three cards from the box. Which of the following events is the most probable one (has the greatest probability)?

A) The three cards are of the same colour.
B) The three cards, regardless of their colours, have numbers 1, 2, and 3.
C) The three cards are of three different colours.
D) The three cards have the same number on them.
E) None of the above, the four previous events have the same probability.

End of Problems

Bonus Problems

Bonus 1: *A mathematically skilled spider spins a web and some of the strings have lengths as shown in the picture. If x is an integer, determine the value of x.*

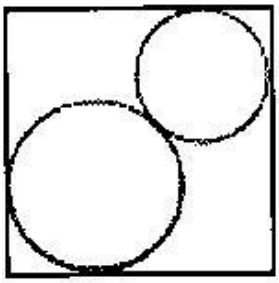


A) 11 *B)*13 *C)* 15 *D)* 17 *E) 19*

Bonus 2: *There are several necklaces in a safe deposit. All the necklaces have the same number of diamonds (at least two diamonds in each necklace). If the number of diamonds in the safe deposit would be known, then the number of the necklaces would also be certainly known. There are more than 200 and less than 300 diamonds in the safe. How many necklaces are there?*

A) 16 *B) 17* *C) 19* *D) 25* *E) other answer*

Bonus 3: *Two circles have their centres on one of the diagonals of a square. They touch each other and the sides of the square, as shown. The square has a side length of 1 cm. What is the sum of the lengths of the two radii of the circles, in centimetres?*



A) $\frac{1}{2}$ *B)* $\frac{1}{\sqrt{2}}$ *C)* $\sqrt{2}-1$ *D)* $2-\sqrt{2}$ *E) It depends on the ratio of the two radii*