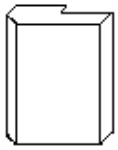
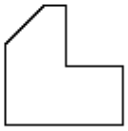
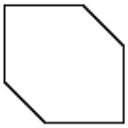
European contest - game
"Math Kangaroo" 2003
Grades 11 and 12

Part A: Each correct answer is worth 3 points.

1) When going to Rimini by train, Lisa sat in the 7^{th} carriage from the front, and Marco sat in front of her, in the 6^{th} carriage from the back, with one carriage between Lisa's and Marco's. How many carriages were there in the train?
A) 15 B) 14 C) 13 D) less than 13 E) not determined

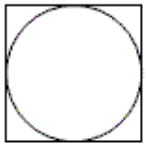
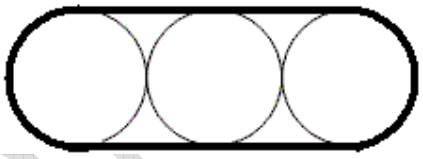
2) Which of the following shapes does correspond to the upper face of the solid on the picture?




A) B) C) D) E)

3) The area of the square in the picture is **a** and the area of each of the circles is **b.** What is the area enclosed by the thick line?

A) 3b B) 2a+b C) a+2b D) 3a E) a+b

4) Alan was calculating the volume of a sphere, but in the calculation he mistakenly used the value of the diameter instead of the radius of the sphere. What should he do with his result to get the correct answer?
A) Divide it by two. B) Divide it by four.
C) Divide it by six. D) Divide it by eight.
E) Divide it by sixteen.

5) $2^{n+2003} + 2^{n+2003} =$

A) 2^{n+2004} B) $2^{2n+4006}$ C) $4^{2n+4006}$ D) $4^{2n+2003}$ E) 4^{n+2003}

6) For which of the following settings does a uniquely determined triangle ABC exist?

A) AB = 11cm, BC = 19cm, CA = 7cm
B) AB = 11cm, BC = 6cm, $\angle BAC = 63°$
C) AB = 11cm, CA = 7cm, $\angle CBA = 128°$
D) AB = 11cm, $\angle BAC = 63°$, $\angle CBA = 128°$
E) For none of them.

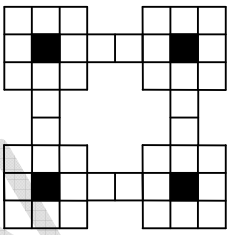
7) The average number of students accepted by a school in the four years 1999 – 2002 was 325 students per year. The average number of students accepted by the school in the five years 1999 – 2003 is 20% higher. How many students did this school accept in 2003?
A) 650 B) 600 C) 455 D) 390 E) 345

8) The set of all values of the parameter m, for which the curves $x^2+y^2=1$ and $y=x^2+m$ have exactly one common point is

A) {-5/4, -1, 1} B) {-5/4, 1} C) {-1, 1} D) {-5/4} E) {1}

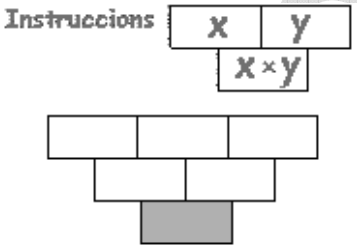
Part B: Each correct answer is worth 4 points.

9) How many possibilities there exist to cover completely all white fields of this composite desk with the usual domino stones 1 x 2?



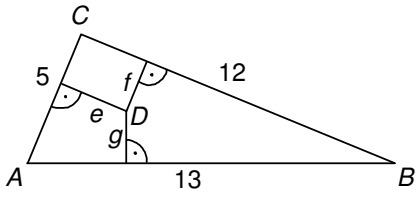
A) 8 B) 16 C) 32
D) 64 E) 100

10) We construct a numerical triangle, with integer number greater than 1 in each cell, following the instructions shown below. What of the numbers pointed out at the answers cannot be placed at the shaded cell?



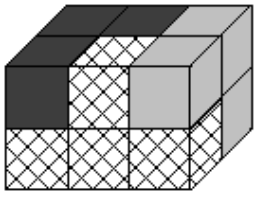
A) 154 B) 100 C) 90 D) 88 E) 60

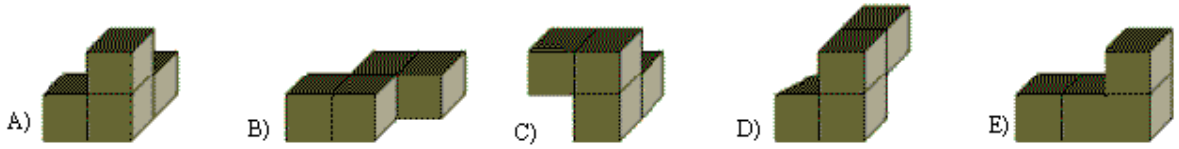
11) Let *ABC* be a triangle with area 30. Let *D* be any point in its interior and let *e*, *f* and *g* denote the distances from *D* to the sides of the triangle. What is the value of the expression $5e + 12f + 13g$?



A) 120 B) 90
C) 60 D) 30
E) It is not possible to determine the value without knowing the exact location of *D*.

12) Using 3 bricks each consisting of 4 little cubes a rectangular parallelepiped has been built (see picture). The crosshatched brick is completely visible, both others are partly visible. Which brick is the dark one?





13) Two white and eight grey seagulls were flying over a river. Suddenly, they all randomly sat down at the bank forming a line. What is the probability that the two white seagulls sit side by side?

A) $\frac{1}{5}$ B) $\frac{1}{6}$ C) $\frac{1}{7}$
D) $\frac{1}{8}$ E) $\frac{1}{9}$

14) $\sqrt{1+2000\sqrt{1+2001\sqrt{1+2002\sqrt{1+2003\times 2005}}}}$ =

A) 2000 B) 2001 C) 2002 D) 2003 E) 2004

15) 12, 13 and 15 are the lengths (perhaps not in order) of two sides of an acute-angled triangle and of the height over the third side of triangle. Find the area of the triangle.

A) 168 B) 80 C) 84 D) $6\sqrt{65}$ E) the area is not uniquely determined

16) A computer is printing a list of the seventh powers of all natural numbers, i. e. the sequence 1^7, 2^7, 3^7, ... etc. How many terms of this sequence are there between the numbers 5^{21} and 2^{49}?

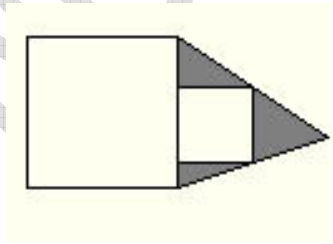
A) 13 B) 8 C) 5 D) 3 E) 2

Part C: Each correct answer is worth 5 points.

17) We know that 10^n+1 is a multiple of 101 and n is a 2-digit number. What is the largest possible value of n?

A) 92 B) 94 C) 96 D) 98 E) 99

18) The diagram shows two squares: one has sides 2m in length and the other has side 1m in length. What is the area of the shaded zone?



A) $1m^2$ B) $2m^2$ C) $2\sqrt{2}$ m^2 D) $4\ m^2$ E) It depends of the position of two squares

19) The sum $100^2-99^2+98^2-\ldots+2^2-1^2$ equals

A) 2002; B) 2020; C) 4040; D) 5050; E) 8008

20) $\left(a+\frac{1}{a}\right)^2=6$, a>0, $a^3+\frac{1}{a^3}$ =

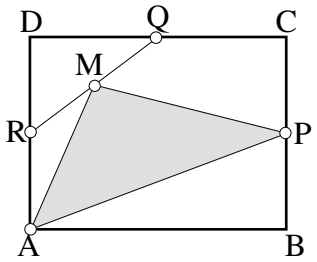
A) $4\sqrt{6}$ B) $3\sqrt{6}$ C) 6 D) $5\sqrt{6}$ E) $6\sqrt{6}$

21) Let A and B be positive integers, such that A> B>1, and A, B, A-B, A+B are all prime. Then S=A+B+(A-B)+(A+B)

A) is even B) is a multiple of 3 C) is a multiple of 5
D) is a multiple of 7 E) is prime

22) In a rectangle ABCD, let P, Q and R be the midpoints of sides BC, CD and AD, respectively, and let M be the midpoint of segment QR. Which fraction of the area of ABCD does triangle ΔAPM cover?

A) 1/4 B) 1/6 C) 3/8
D) 1/3 E) 5/16



23) A sequence $(a_n)_{n\geq 0}$ is defined in the following way :

$a_0=4$

$a_1=6$

$a_{n+1}=(a_n)/(a_{n-1})$, $n\geq 1$.

Then a_{2003} is equal to:

A) 3/2 B) 2/3 C) 4 D) 1/4 E) 1/6

24) Let f be a polynomial such that $f(x^2 + 1) = x^4 + 4x^2$. Determine $f(x^2 - 1)$.

A) $x^4 - 4x^2$ B) x^4 C) $x^4 + 4x^2 - 4$ D) $x^4 - 4$
E) another answer

Contest Game
"Math Kangaroo"
March 29, 2003

Answers
Grade 11-12

1	A B C **D** E	**9**	A **B** C D E	**17**	A B C **D** E
2	A B C D **E**	**10**	**A** B C D E	**18**	**A** B C D E
3	A **B** C D E	**11**	A B **C** D E	**19**	A B C **D** E
4	A B C **D** E	**12**	**A** B C D E	**20**	A **B** C D E
5	**A** B C D E	**13**	**A** B C D E	**21**	A B C D **E**
6	A B C D **E**	**14**	A **B** C D E	**22**	A B C D **E**
7	**A** B C D E	**15**	A B **C** D E	**23**	A **B** C D E
8	A B C D **E**	**16**	A B C D **E**	**24**	A B C **D** E